

# Hybrid Vehicle Energy Model: Analysis of Gas-Electric Hybrid Fuel Efficiency

Greg Zilberbrant, McMaster School of Engineering Practice, August 2008

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## Abstract

The lack of an external energy source for electrical power for a gas-electric hybrid requires the balance between usage and regeneration of the battery. This balance is critical to the fuel efficiency of the hybrid vehicle. Fuel efficient vehicles continue to emerge as a tool to reduce individual, corporate, and societal environmental impact – the driving criteria under which gas-electric hybrids are more fuel efficient than small engine vehicles must be understood if the goal of reduced environmental impact is achieved. In this paper, an analysis is performed on the entire operational spectrum of a vehicle to develop a model that can be applied to gas-electric hybrids. The resulting model can be applied to mild or full gas-electric hybrids to establish true fuel efficiency based on the energy balance between battery charge and depletion modes.

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## Introduction

The fuel efficiency trend in North America over the past eight year has been towards the gas-electric hybrid. Sixteen models are available for the 2008 model year - a drastic shift from the initial release of the Honda Insight and Toyota Prius in 1999 and 2000, respectively.

There is credible evidence that a hybrid vehicle will require less gasoline (or diesel) to perform under the same conditions as its non-hybrid counterpart. This is seen in the Natural Resources Canada Fuel Consumption Guide, the United States Environmental Protection Agency (U.S. EPA) Fuel Economy Guide, as well as on road test vehicles [2][3][18]. The gas-electric hybrids have a distinctive advantage over its non-hybrid counterparts that can only be achieved with an energy recovery system: regenerative braking. Regenerative braking allows energy to be recovered via an electric generator and stored in a battery. This additional energy is used to supplement power needs of the vehicle during high

demand driving, such as acceleration, in a more efficient manner.

The emergence of fuel efficient vehicles is not limited only to gas electric hybrids but smaller vehicles in general. The Smart Fortwo is the iconic example of a small vehicle designed for the individual commuter (or double occupancy maximum as the name would suggest). With a one litre engine operating on three cylinders, the Smart Fortwo is comparable in fuel efficiency to all hybrid vehicles for city driving and is surpassed only by the Toyota Prius and Honda Civic Hybrid for highway driving based on the 5-cycle U. S. EPA testing [3].

The variation in test methods used by Canada and the United States further complicates the analysis. The fuel efficiency for the Honda Civic Hybrid and Toyota Prius differs by 48 percent between the Canadian and American publication. It is obvious that variation in testing procedure results in a variation in final fuel efficiency calculation, however, the information produced by Natural Resources Canada and the U.S. EPA is used by consumer and scientists to understand the fuel use of vehicles. Both agencies, as well as the

manufacturers and fuel testing laboratories, do not claim that the fuel economy determined by their methods is representative of an individual driver's results. Testing is performed in a controlled environment in order to allow the greatest repeatability of testing [4]. This, in turn, produced information that is ideal for relative comparison. However, this does little to provide the individual driver with performance results based on different driving conditions and needs.

### Gas-Electric Hybrid Functionality

Commercially available hybrids are engineered in two formats: mild and full hybrids. The mild and full hybrids have significant similarities in that both have the following features:

1. Auto Idle Off (engine off during idling)
2. Regenerative Breaking (energy is recovered via generator during deceleration)
3. Electric Assist (battery provides additional power during high demand driving)

The full hybrid differs in that it can operate solely on electrical power in scenarios where the combustion engine is less efficient [5]. The Auto Idle Off feature can be achieved in conventional vehicles with modification or at the will of the driver to manually perform the function. Performing the task manually, however, may be tedious for the driver and taxing on the starter.

Regenerative breaking is the energy recovery benefit of hybrid vehicles. The gas-electric hybrids draw all energy from the internal combustion engine (ICE). The net benefit in fuel efficiency is through the recovery of potential energy losses and the reduction of energy conversion inefficiencies. The former is strictly through regenerative breaking while the latter uses the energy recovered through regenerative breaking and steady speed driving when ICE energy conversion is optimal.

Due to this balance of energy recovery and depletion the driving patterns have specific limitation in order to maintain hybrid performance. Figure 1 shows the functional flow of a mild and full hybrid. As noted earlier, the only distinction is the “Electric Only Motion” that is available with the full hybrid denoted by the dashed lines. Both diagrams represent the potential shift in functionality of the vehicle as it operates. The functions of the vehicle are divided into three groups referred to as the “Driving Range”. “Driving Range 1” (D1) and “Driving Range 3” (D3) represent electrical energy depletion. “Driving Range 2” (D2) represents electrical energy recovery. In order for a hybrid vehicle to function as such (i.e. sufficient electrical energy is available when required) then the net energy recovered from D2 must be equal to or greater than the gross electrical energy required during D1 and D3 to supplement ICE energy (Equation 1).

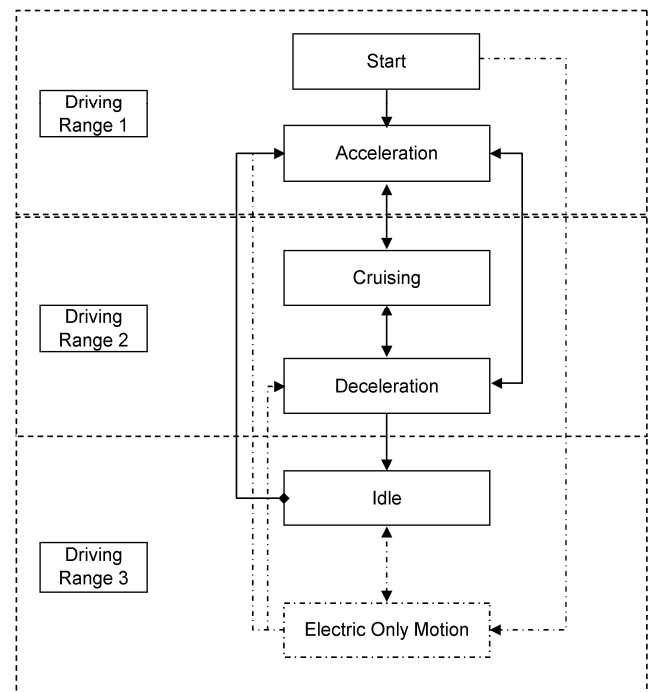


Figure 1: Process Flow for Mild and Full Hybrids

$$Net\ Energy_{D2} \geq Gross\ Energy_{D1+D3} \quad (Equation\ 1)$$

**Note:** Equation 1 represents the energy balance at the battery where incoming efficiency losses have been accounted for.

### Functionality Limitations

The energy balance presented in Equation 1 is crucial to the fuel efficiency of hybrid vehicles. The fuel efficiency benefits of operating in an electrically demanding mode (at lower gas consumption) must be balanced with regenerative energy driving in order to meet driving demands. Only 3 of the 16 hybrid models available in 2008 have smaller engines than their non-hybrid counterparts. The operation of the remaining 13 models without hybrid functionality would have no fuel efficiency benefit simply from having a smaller engine [19]. Without the battery energy providing additional power, the engine is also required to meet the demands of the heavier vehicle (hybrids outweigh their non-hybrid counterparts by an average of 7.4%) affecting its driveability [6-15]. The limitations are critical to understanding driving ranges outside which the hybrid is simply an underpowered vehicle.

Figure 2, 3 and 4 represent the hybrid functionality limitation for acceleration, cruise, and electric only driving, respectively. The Start and Idle functions, presented in Figure 1 but not detailed in this section, have only a battery power availability limitation. Insufficient energy from a battery (whether a Nickel-Metal Hydride or Lead-Acid) will fail to start an ICE. Idle mode energy is also dependent only on the available stored electrical energy. However, if the available energy is insufficient to operate the accessories and re-engage the ICE once in motion then the hybrid will switch to ICE mode while idling. The limitation of the idle functionality is represented in part by Figure 4. Deceleration is limited by the battery energy level prior to deceleration. Battery energy levels are generally maintained between 40-60% to ensure its longevity [16]. If the battery is fully charged then it cannot accept additional energy. Outside of this scenario, deceleration has no functionality limitations.

### Acceleration

The primary limitation for acceleration is the availability of energy in the battery. This criterion is

the primary requirement for any electric assisted or electric only functionality. The following defines the functionality of acceleration:

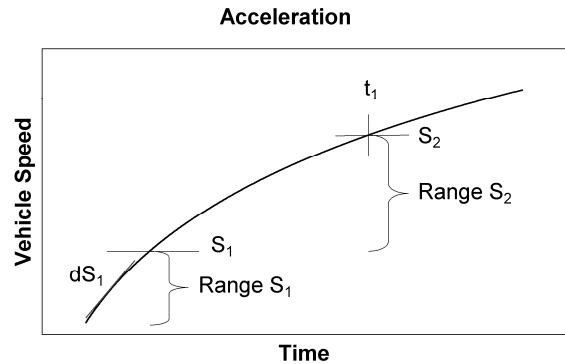


Figure 2: Acceleration Functionality Limitations

Hybrid acceleration can occur if:

1.  $t_1$  is not met prior to  $S_2$
2.  $dS/dt \leq dS_1$  (full only)

where (for Figure2):

$S_1$  = electric only speed limit

$dS_1$  = acceleration that can be met by electric only function

$t_1$  = battery energy limit (empty)

$S_2$  = end of acceleration

Range  $S_1$  = electric only operating range

Range  $S_2$  = combined ICE and electric operating range

Mild hybrids would operate the same through Range  $S_1$  and Range  $S_2$  as the electric only function represented by Range  $S_1$  does not exist. Full hybrids balance the use of energy for Range  $S_1$  with the need for supplemental power in Range  $S_2$ . As the battery becomes depleted through electric only operation in Range  $S_1$ ,  $t_1$  approaches zero until the following scenarios occur in progression: there is insufficient energy available to complete acceleration with battery assist (condition 1 is met); there is insufficient energy available to begin acceleration above electric only speed ( $t_1$  intersects  $S_1$ ); and finally there is insufficient power to meet electric only operations (battery is depleted). Condition 2 does not eliminate hybrid functionality

but eliminates the electric only function forcing the full hybrid to function as a mild hybrid.

### Cruise

In cruise mode, criteria must be met in order to recharge the battery. Unlike all other functionality limitations for hybrids, when this criteria is not met, hybrid functionality is not immediately effected. There is no direct fuel efficiency benefit to meeting cruise criteria. However, meeting these limitations for a specific time period allows the vehicle's battery to be recharged so that the primary hybrid criterion (sufficient battery energy availability) is met for all electric assist and electric only functions. Figure 3 represents the criteria that must be met for battery recharging during cruise function.

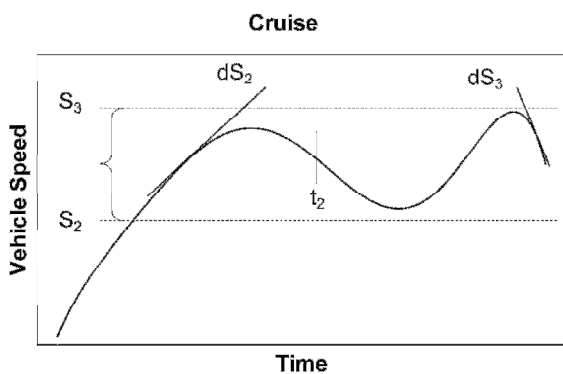


Figure 3: Cruise Functionality Limitations

Battery recharging can occur if:

1. Vehicle speed  $\geq S_2$
2.  $dS/dt \leq dS_2$
3.  $t_2$  is not met

where (for Figure3):

- $S_2$  = minimum speed for cruise mode
- $S_3$  = maximum speed for efficient operation
- $dS_2$  = acceleration that is efficiently met by ICE
- $dS_3$  = deceleration mode activated (battery recharge continue is dec. mode)
- $t_2$  = battery energy limit (full charge)

Mild and full hybrids would operate identically in cruise mode. Vehicle speed above  $S_3$

is considered to be above a safe operating mode where fuel efficiency would decrease significantly as a result of increased drag. The overall fuel efficiency benefit of recharging during cruise mode may be negated by excessive fuel requirement when operating at speeds above  $S_3$ . It is therefore expected that the driver will not operate the vehicle in an aggressive manner and not exceed  $S_3$  unless required for passing.

### Electric Only

The electric only mode for mild and full hybrids is represented with a solid and dashed line, respectively, in Figure 4. For a mild hybrid, the maximum speed for electric only mode is zero (idling). Full hybrids have a programmed maximum electric only operating speed ( $S_1$ ).

The functionality of the electric mode is limited by the primary hybrid criterion as well as maximum acceleration or deceleration (at less than maximum electric only vehicle speed) without meeting the requirement for a mode switch.

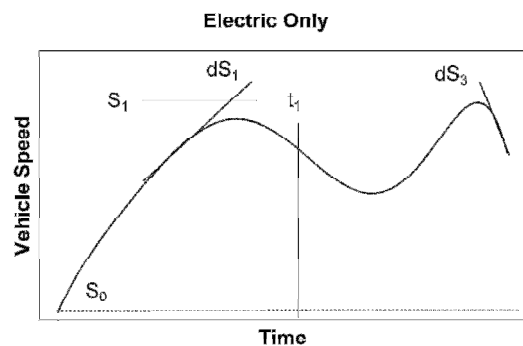


Figure 4: Electric Only Functionality Limitations

Electric only can occur if:

1.  $t_1$  is not met
2.  $dS/dt \leq dS_1$  (full only)
3. Vehicle speed =  $S_0$  (mild only);  
Vehicle speed  $\leq S_1$  (full only)

where (for Figure4):

- $S_0$  = auto idle off operation (vehicle operating but motionless)
- $S_1$  = maximum speed for electric only operation (full only)

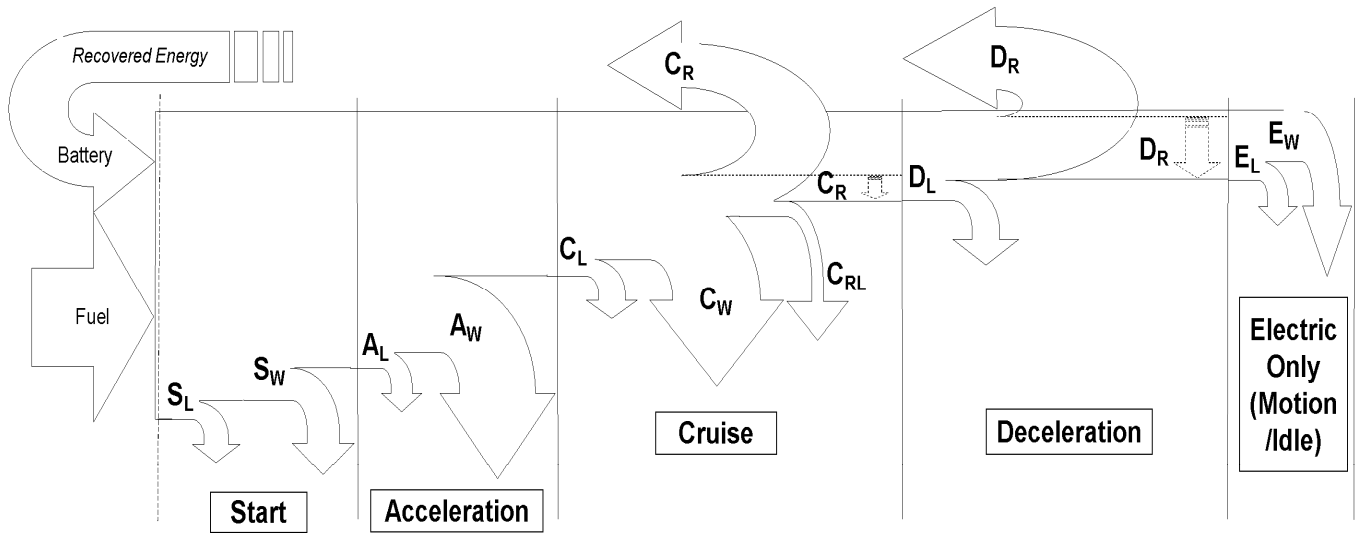
$dS_2$  = acceleration mode activated (battery assist required)  
 $dS_3$  = deceleration mode activated (battery recharge)  
 $t_1$  = battery energy limit (empty)

## Energy Flow

As stated earlier, the fuel efficiency advantage of gas-electric hybrids is realized through the recovery of potential energy losses (regenerative braking) and the reduction of energy conversion inefficiencies. Energy losses average 80% for ICE only vehicles while the combined energy

efficiencies of hybrid components reduce the losses to approximately 40% [17].

The previous section outlined the details of the functionality criteria that must be met in order for the hybrid vehicle to operate as it is designed (not reverting to an ICE solely powered vehicle). With these criteria met, the energy flow of a hybrid vehicle is analyzed to understand the potential losses and recoveries in each driving mode. Figure 5 shows a hybrid vehicle with a fully charged battery and fuel tank as it experiences all modes of transportation.



**Figure 5:** Energy Flow for Hybrids

### Start

The energy requirements to obtain a preferred engine temperature are represented by  $S_W$  with the associated losses represented by  $S_L$ . The energy requirements for start mode are met by the ICE and vary depending on temperature and soak time (amount of time ICE is not engaged) [1].

Temperature variation is a significant factor for fuel efficiency, specifically in Canada. The temperature adjustment for fuel efficiency is addressed in the equations presented for the model.

### Acceleration

Acceleration has a mixed loss, represented by  $A_L$ , of ICE and electric energy. The demand for power is met by both energy sources but varies with respect to speed. The energy impact of  $A_W$  represents the average energy requirement for acceleration after related losses ( $A_L$ ).

Energy demands for acceleration can vary significantly, especially at higher speeds and quicker acceleration. It is therefore assumed that the driver will operate the vehicle in a safe mode and

greater acceleration will be limited to passing requirements only (as mentioned previously).

*Cruise*

Cruise mode has the potential to recharge the electric battery efficiently as steady speed operation is the optimal operating mode for a combustion engine. The ICE operational components are

represented by  $C_W$  (energy to drivetrain) and  $C_L$  (ICE efficiency losses). The energy components of battery recharging are represented by  $C_R$  and  $C_{RL}$ .  $C_R$  represents the net energy that is recovered by the battery with the transfer losses accounted for by  $C_{RL}$ . Therefore,  $C_R$  represents the energy available at the battery as a result of regeneration during cruise mode. Once the battery is fully charged, cruise mode would be represented by  $C_W$  and  $C_L$  only.

Operating Mode	Hybrid Type	Criteria	Result	Usage		Battery Energy (from Figure 5) Net Recovery/Gross Requirement
				ICE %	EM/EG %	
Start	M/F	standard starter system	ICE mode	100	-	
Acceleration (A mode)	M	$t_1$ not met and $dS/dt > 0$	ICE/E mode	x	$y_1$	$(A_W + A_L) * y_1$
	F	$t_1$ not met and $dS/dt \geq dS_1$	ICE/E mode	x	$y_1$	
	M/F	$t_1$ met	ICE mode	100	-	
	F	$dS/dt < dS_1$	E mode	md	md	
Cruise (C mode)	M/F	vehicle speed $\leq S_3$ and $dS_3 \leq dS/dt \leq dS_2$ and $t_2$ not met	Recharge	100	$y_2$	$C_R * y_2$
	M/F	vehicle speed $> S_3$	ICE mode	100	-	
	M/F	$dS/dt > dS_2$	A mode	md	md	
	M/F	$t_2$ met	ICE mode	100	-	
	M/F	$dS/dt < dS_3$	D mode	md	md	
Deceleration (D mode)	M/F	$dS/dt < 0$	Recharge	-	100	$D_R$
	M/F	$dS/dt \geq 0$	A or E mode	md	md	
Electric Only (E mode)	M	$t_1$ not met and vehicle speed = 0	E mode	-	100	$E_W + E_L$
	F	$t_1$ not met and vehicle speed $\leq S_1$ and $dS_3 < dS/dt < dS_1$	E mode	-	100	
	M/F	$t_1$ met	ICE mode	100	-	
	M/F	$dS/dt \leq dS_3$	D mode	md	md	
	M	vehicle speed $> 0$	A mode	md	md	
	F	vehicle speed $> S_1$	A or C mode	md	md	
	F	$dS/dt \geq dS_1$	A mode	md	md	

M/F = mild and full hybrid criteria  
M = mild hybrid only criteria  
F = full hybrid only criteria  
ICE = internal combustion engine  
EM/EG = electric motor/electric generator  
md = mode dependent

**Table 1:** Hybrid Driving Model

### Deceleration

The energy balance for deceleration (ie. regenerative braking) is simply related to energy recovery. As with cruising, the energy recovered is represented as the net energy available at the battery.  $D_R$  is the recovered net energy with losses accounted for with  $D_L$ .

### Electric-Only (Motion/Idle)

The efficiency losses, represented by  $E_L$ , are the lowest in electric-only mode. Whether in idle mode or in electric-only motion, the efficiency losses represented by  $E_L$  are much lower relative to the actual energy used to do work (represented by  $E_W$ ). As mentioned earlier, the efficiency losses for an electric motor are much lower than those for a combustion engine which makes this operating mode the most energy efficient.

### Hybrid Driving Model

The analyses presented in the previous sections of this paper have been summarized in Table 1. The culmination of this model is a full interpretation of the functional modes of a gas-electric hybrid vehicle. Equation 1 that was presented previously as;

$$Net\ Energy_{D2} \geq Gross\ Energy_{D1+D3} \quad (Eq. 1)$$

can be rewritten as;

$$(A_W + A_L)*y_I + E_W + E_L \geq C_R*y_2 + D_R \quad (Eq. 2)$$

Equation 2 represents the ideal driving scenario in that a sufficient amount of electric energy is recovered through regenerative braking and cruising to meet electrical energy requirements for acceleration and electric mode. In the case where

equation 2 is not true, the following equation should be used;

$$\begin{aligned} [(A_W + A_L)*y_I + E_W + E_L] - [C_R*y_2 + D_R] \\ = Energy\ from\ ICE\ only\ use \end{aligned} \quad (Eq. 3)$$

Finally, to evaluate the fuel efficiency benefit of a hybrid vehicle, Equation 4 should be used as follows;

$$ICE\ only\ \% = \frac{Energy\ from\ ICE\ only\ use \times 100}{[(A_W + A_L)*y_I + E_W + E_L]} \quad (Eq. 4)$$

$$hybrid\ use\ \% = 100 - ICE\ only\ \% \quad (Eq. 5)$$

The hybrid function in Equation 5 represents the fraction of operation (relative to energy use) that a vehicle will operate at the lab tested fuel efficiency where battery depletion is not a factor. Equation 4 represents the fraction where battery energy is insufficient to meet vehicle demand. The combination of Equation 4 and Equation 5, shown in Equation 6, represents the fuel efficiency (FE) of the vehicle where *ICE FE* can be a tested or back calculated value the *lab tested FE*.

$$FE = (ICE\ only\ \% *ICE\ FE + hybrid\ use\ \% *lab\ tested\ FE)/100 \quad (Eq. 6)$$

Laboratory testing procedures for the fuel efficiencies being used in Equation 6 should be reviewed and adjust for temperature variation in equations 2 through 5 as required. It is not recommended that more than two temperature averages (lab temperature and average winter temperature) are used in this analysis as further refining will take the temperature adjustment beyond the precision the model is intended for.

## Conclusion and Discussion

The hybrid fuel efficiency model presented in this paper is intended to assist researchers in developing algorithms and simplified decision criteria. These tools can, in turn, be coupled with fuel efficiency guides and websites produced by government agencies to help consumers make environmentally and economically beneficial vehicle purchases.

Increase cost of crude oil and refining costs will allow the fuel efficient vehicle market to flourish in the foreseeable future. As a majority of the greenhouse gas impact of vehicles is associated with gas consumption, even for significantly more fuel efficient vehicles [17], decision criteria can be used by consumers to make the most environmental (ie. fuel efficient) choice for their driving patterns – whether such a choice results in a hybrid or small engine compact vehicle.

Further research on this topic must come in the form of individual vehicle evaluation. Functionality criteria and efficiency formulae should be evaluated for individual vehicles and driving modes. This information is the next critical step in creating an effective decision making tool for consumers, corporations, and government agencies.

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